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Strong Electromagnetic Field

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by

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ABSTRACT

The collisional energy loss of an electron undergoing forced oscillation in an electromagnetic field behaves quite differently in the low and high intensity limits. In the case where the thermal velocity v_t is much larger than $v_0 = eE_0/m\omega$ the rate at which the electron transfers energy to the random motion of the medium is proportional to v_0^2. It is shown that in the case of an electromagnetic field $v_0 \gg v_t$ the rate of transfer is much slower, and actually decreases with the strength of the field.

Suppose an electron is subject to a strong electromagnetic (laser) field

$$E_x = E_0 \cos \omega t . \tag{1}$$

Then, in the absence of coupling with the medium (ions and background electrons), and neglecting relativistic effects, the electron will execute simple harmonic motion with velocity

$$v_x = v_0 \sin \omega t \tag{2}$$

where $v_0 = eE_0/m\omega$ is the "driven velocity".

How will the large electron energy which is in the motion (2), be transmitted to the medium? I shall disregard here the plasma instabilities although I believe that in reality these are probably the most important mechanism. I shall confine myself to collisions with individual atoms, ions or background electrons, both elastic and inelastic. The elastic collisions in the field of an atom or ion are "inverse bremsstrahlung". The atoms or ions may be assumed to be at rest for our purposes.

The transport elastic cross section between an electron of velocity v and an ion of charge Z is

$$\sigma_t = 4\pi \frac{Z^2 e^4}{m^2 v^4} \ln \frac{1}{\theta_{min}} \tag{3}$$

where

$$\theta_{min} = \frac{2Ze^2}{mv^2 b_{max}} \tag{3a}$$

and b_{max} is the maximum impact parameter, approximately half the distance between ions. The ℓn in (3) is of the order 10, its exact value and dependence on v are unimportant. For $v = c$ and $Z = 1$,

$$\sigma_t \approx 10^{-23} \text{ cm}^2 . \tag{4}$$

The rate of collisions per unit time in a medium containing N_a atoms per cm^3 is

$$\gamma = N_a \sigma_t v = A v^{-3} \tag{5}$$

where A can be obtained from (3).

Clearly, the electron moving according to (2) will collide mostly when its velocity v is small, i.e. close to the turning points of its motion. By these collisions, it will be deflected, and will acquire a random velocity in the yz-plane, the energy for this being drawn from its ordered velocity in the x-direction. We call the random motion in the yz-plane the "thermal" motion, with velocity v_t. Then the energy gained per unit time by the thermal motion, at the cost of the directed motion, will be roughly proportional to

1

$$\frac{d(v_t^2)}{dt} = \gamma v_x^2 = A \frac{v_x^2}{(v_x^2 + v_t^2)^{3/2}} . \qquad (6)$$

The first part of this equation states that γ is the rate at which kinetic energy in the x-direction is converted into random motion kinetic energy. In the second part, we have used (5), and set $v = (v_x^2 + v_t^2)^{1/2}$.

We insert (2) into (6) and integrate over a quarter period of the x-motion,

$$(\Delta v_t^2)_{1/4} = A \int_0^{\pi/2\omega} \frac{v_o^2 \sin^2 \omega t}{(v_o^2 \sin^2 \omega t + v_t^2)^{3/2}} dt . \qquad (7)$$

If $v_t \ll v_o$, the integral can be evaluated approximately and gives

$$(\Delta v_t^2)_{1/4} = \frac{A}{v_o \omega}\left(\ln \frac{v_o}{v_t} + \ln 4 - 1\right) . \qquad (8)$$

Therefore the secular change of the random energy is

$$\left(\frac{d(v_t^2)}{dt}\right)_{Av} = \frac{(\Delta v_t^2)_{1/4} \, 2\omega}{\pi}$$

$$= \frac{2}{\pi} \frac{A}{v_o}\left(\ln \frac{v_o}{v_t} + \ln 4 - 1\right) \quad (v_o \gg v_t) . \qquad (9)$$

If, on the other hand, $v_o \ll v_t$, then we have the usual case of inverse bremsstrahlung, and the average of (6) gives simply

$$\left(\frac{d(v_t^2)}{dt}\right)_{Av} = A \frac{v_o^2}{2v_t^3} = \frac{1}{2} \gamma_t v_o^2 \quad (v_o \ll v_t) . \qquad (10a)$$

The correct value, obtained by elementary considerations, is

$$\left(\frac{d(v_t^2)}{dt}\right)_{Av} = \gamma_t v_o^2 = \frac{A v_o^2}{v_t^3} \quad (v_o \ll v_t) . \qquad (10)$$

The difference between (9) and (10) is striking. In the usual case (10), the energy transferred to random motion is proportional to v_o^2, hence to the energy of the incident electromagnetic field. For a strong field, (9) states that the energy absorbed goes inversely as v_o; the stronger the field, the less energy is absorbed. Further, (9) is nearly independent of the thermal energy already present, v_t, while (10) goes as v_t^{-3}. Clearly, as long as the thermal velocity is less than the driven velocity, much less energy is absorbed from the incident electromagnetic wave than the "classical" formula (10) would indicate.

It is easy to calculate the rate at which the thermal velocity approaches the driven velocity. Keeping v_o constant, and using (5), (9) may be written

$$\frac{d}{dt}\left(\frac{v_t^2}{v_o^2}\right) = \frac{2}{\pi} \gamma(v_o) (\quad) \qquad (11)$$

where $\gamma(v_o) = A v_o^{-3}$ and ( ) is the parenthesis in (9). Using $v_o = c$, and (4), and setting ( ) = 1 on the average, we get

$$\frac{d}{dt}\left(\frac{v_t^2}{v_o^2}\right) = 2 \cdot 10^{-13} N_a . \qquad (11a)$$

Assuming $N_a = 10^{21}$, a typical critical density for laser light, the time required for v_t to reach v_o is 5 nanoseconds, i.e. quite long.

Relativistic generalization should be easy, and we must expect that the time required for energy absorption is even longer. Of course, once $v_t = v_o$, the usual absorption rate (10) becomes valid, and the thermal velocity can become much larger than the driven one if the electromagnetic wave continues to act.

INELASTIC COLLISIONS

Energy can be lost, both from the ordered and the random motion of the electrons, also by inelastic collisions. We estimate the number of ionizing collisions by taking the stopping power to be

$$2 \text{ MeV per g/cm}^2 \qquad (12)$$

which corresponds about to minimum ionization in D_2.

Taking again $N_a = 10^{21}/cm^3$, we have $\rho = 3 \cdot 10^{-3}$ g/cm^3. Taking then 30 ev for the energy required to make an ion, we get 200 ions per cm. Each ionizing collision creates a new electron which again acquires the velocity (2). After traversing ℓ cm, the number of electrons will be $e^{200\ell}$. Complete ionization will be achieved if the number of electrons is e^{50}; for this the electrons need to travel about $\frac{1}{4}$ cm. At velocity of light, this requires about 10 picoseconds.

Excitation of discrete states is possible if the atoms are not completely stripped which is possible for heavy materials. For neutral atoms, we take the energy loss by inelastic collisions to be again given by (12). The density is roughly

$$\rho = 3 \cdot 10^{-24} N_a Z \qquad g/cm^3 \qquad (13)$$

hence the inelastic energy loss per unit time, for velocity c:

$$\left(\frac{dE}{dt}\right)_{inel} = 2 \cdot 10^6 \cdot 3 \cdot 10^{-24} N_a Z c$$

$$= 2 \cdot 10^{-7} N_a Z \text{ ev/sec.}$$

To compare with (11a), we write this in the form

$$\frac{1}{mc^2}\left(\frac{dE}{dt}\right)_{inel} = 4 \cdot 10^{-13} N_a Z . \qquad (14)$$

It should be remembered, however, that for atoms of charge Z, Eq. (3) has a factor Z^2, so (11a) should be replaced by

$$\frac{d}{dt}\left(\frac{v_t^2}{v_o^2}\right)_{Coul} = 2 \cdot 10^{-13} N_a Z^2 . \qquad (11b)$$

Only for large Z can we expect the atoms to be not completely stripped, and if they are partially stripped, Z in (14) should be replaced by Z minus the number of bound electrons in each atom. Thus in general, the Coulomb collisions (inverse bremsstrahlung) discussed in the main part of this report will cause a stronger energy loss from the "driven" motion (and thus from the electromagnetic wave) than the inelastic ones, at least after complete ionization has been achieved.

CONCLUSION

Energy absorption from a very intense electromagnetic wave by ordinary atomic processes is very slow, especially for Z = 1. To achieve rapid absorption, collective instabilities are required.